- **Basis**: A basis of *V* is a set of linearly independent vectors *S* such that *S* spans *V*.
    - Let $S = \{\vec{v}_1, \vec{v}_2, ..., \vec{v}_n\}$. $\forall \vec{v} \in V, \exists! c_1, c_2, ..., c_n : \vec{v} = c_1\vec{v}_1 + c_2\vec{v}_2 + ... + c_n\vec{v}_n$
        - i.e. $c_1, c_2, ..., c_n$ is unique for a basis
        - $c_1, c_2, ..., c_n$ are the **coordinates** of $\vec{v}$ in the basis *S*.
- **Dimension** $\dim(V)$: the dimension of *V* is the cardinality of any basis.
    - **Finite-dimensional**: the basis of *V* is finite
        - A set with cardinality greater than the dimension is linearly dependent.
        - A set with cardinality less than the dimension cannot span *V*.
        - A set with cardinality equal to the dimension is linearly independent iff it is not a spanning set. Otherwise, it is a basis.
        - All bases for *V* have the same number of vectors.
        - If *S* spans *V* but is not a basis (i.e. a spanning set with cardinality greater than the dimension), then *S* can be reduced to a basis by removing linearly dependent vectors.
        - If *S* is linearly independent but does not span *V* (i.e. linearly independent set with cardinality less than the dimension), then a basis can be built by adding linearly independent vectors
    - **Infinite-dimensional**: the basis of *V* is countably infinite
    - Common dimensions:
        - $\dim(R^n) = n$
        - $\dim(P^n) = n + 1$
        - $\dim(M_{mn}) = mn$
- **Standard basis**
    - R^n - $\{\langle 1,0,...,0\rangle, \langle 0,1,...,0\rangle, \ldots, \langle 0,0,...,1\rangle\}$
    - P^n - $\{1, x, x^2, ..., x^n\}$
- **Change of basis**
    - Suppose you have two sets of bases, *B* and *B'*. Then the change in basis matrix from *B* to *B'*, denoted $P_{B \to B'}$, has the columns such that each column contains the coordinates of each vector in *B* in *B'*.
    - Alternatively, you could find perform Gauss-Jordan elimination on the matrix $(B' \quad B)$.
    - Also, note that $P_{B' \to B} = P_{B \to B'}^{-1}$
    - $P_{B \to B'}$ is used to convert coordinates in *B* to coordinates in *B'*.